DISCRETE ELEMENT ANALYSIS OF ROCK FAILURE CHARACTERISTICS UNDER UNIAXIAL COMPRESSION

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The failure process of rock material under uniaxial compression is simulated using discrete element method with parallel bond, in which the spherical particles are bonded together with initial random dense packing. The simulated results indicate that the failure process from initial elastic deformation, crack generation to final breakage can be modeled well with the developed discrete element method. The tensile failure is the major failure mode in the failure process of rock sample. The influences of particle size on the elastic modulus, Poisson's ratio and compressive strength are not obvious. This study will be helpful in revealing the failure mechanism of rock material under uniaxial compression and also in verifying the application of discrete element method into rock mechanics.

INTRODUCTION

Uniaxial compression test of rock material has been widely used in rock engineering to reveal its basic mechanical behaviors because of its simplicity. The elastic modulus, Poisson's ratio and compressive strength of rock material can be determined with the measured stress-strain curve during rock failure process^[1]. The sample scale effect and failure mode have also been investigated^[2-3]. Discrete element method, with the advantage of no complex constitutive relationship needed, has been successfully applied in the numerical simulation of laboratory rock material failure process, and is becoming a useful tool in geomechanics problems such as stability analysis of rock slope^[4-6].

Traditional DEM mainly models the contact and movement of discontinuous material. Recently, DEM has been extended to simulate mechanical behaviors of continuous material by establishing bonding model and its failure criteria between particles^[7-12]. Here, the parallel bond model is adopted to model the breakage of rock material in compressive mode, which acts over a circular cross-section lying between the particles, and can transmit both a force and a moment.

Studies have shown that the elastic modulus and strength decrease with increasing sample size because of the internal micro damage^[13]. In addition, particle size influences the computational results^[14]. The reasonable choice of particle size to eliminate scale effect is an important part in the DEM simulation. Therefore, this paper aims to investigate the change of the compressive strength, elastic modulus and Poisson's ratio of rock material, and the effect of particle size under uniaxial compression using the discrete element model with parallel-bond model.

DISCRETE ELEMENT METHOD FOR ROCK MATERIAL

The failure criteria of the parallel bond for tensile and shearing are shown in Fig.1. Here, K_n and K_s are the normal and tangential stiffness, R_n and R_s are the normal and tangential force, u_{nmax} and u_{smax} are the corresponding maximum normal and tangential displacement when the

bond breaks. The bonding strength is a key parameter influencing the simulation results. Considering the non-uniform and discontinuity of rock material at micro scale, Weibull distribution based probability density function is adopted to analyze mechanical parameters, that is

$$f(x) = \frac{m}{u_0} \left(\frac{u}{u_0}\right)^{m-1} \exp\left(-\left(\frac{u}{u_0}\right)^m\right)$$
(1)

where, *u* is the value of any parameter that meets Weibull distribution, u_0 is the average of all elements, *m* is the shape parameter. Due to the random distribution of the bonding strength, the average of the tensile strength is set as $\sigma_{T0} = 50$ MPa and m = 6, thus tensile bonding strength can be randomly generated. Considering the fact that no bonding effect is produced when the tensile bonding strength is small, random number is chosen between $[0.8 \sigma_{T0}, 1.2 \sigma_{T0}]$ to keep the integrity of the initial sample. The shearing strength is set as 2.5 times of the tensile strength.



Fig1. Tension and shear constitutive relations of bonded particles and failure criteria

The normal and tangential force is calculated through the contact stiffness and overlap between particles in contact. Liner contact model with parallel bond is considered, and the normal stiffness is written as $K_n = \pi DE/4$, where D is the average particle diameter, E is the elastic modulus of rock material. Tangential stiffness is usually determined by the normal one, and we have $K_s = 0.2K_n$.

The sample with size of 100mm×100mm×200mm is generated through particle growing, stabilizing and adding parallel bond. Particle size obeys normal distribution in the range of [0.8D, 1.2D] with $\mu = D$ and $\sigma = 0.25$. The effective cross sectional area is, $A_e = (L - \beta D)(B - \beta D)$, where L and B are the length and width of the sample, β is correction coefficient and set as 1.0. Other main computational parameters are listed in Table 1.

DEM SIMULATION RESULTS AND ITS FAILURE CHARACTERISTICS ANALYSIS

The stress-strain curve of the rock sample is shown in Fig 2. The maximum stress is considered to be the compressive strength of rock material, and the slope of the linear part of the curve is treated as the elastic modulus. Hence, the compressive strength of the sample is 217.4 MPa, the corresponding maximum strain is 5.2×10^{-3} , and the elastic modulus is 49.4 GPa.

The stress-strain curve shows that the compressive failure process of the rock sample can be divided into four stages: initial linear elastic (*O*-*A*), crack generation (*A*-*B*), softening (*B*-*E*) and final failure (*F*-*I*). During the failure stage, inclined crack at 45° to the loading direction appears as shown in Fig 3. With further loading, breakage aggravates, the internal stress approaches zero

and the sample loses its bearing capacity. In addition, the sample is in the linear elastic stage when the corresponding strain is smaller than 2.5×10^{-3} from Fig 2, and the Poisson's ratio in this stage is can be taken as the Poisson's ratio of the sample.



Fig.2 Stress-strain curve during uniaxial compression of rock sample



Fig.3 Failure mode of rock sample under uniaxial compression

Definition	Symbol	Value
Density	ρ	2630 kg/m3
Sample size	$a \times b \times c$	100mm×100mm×200mm
Average size	D	7mm
Particle elastic modulus	Ε	150GPa
Friction coefficient between particles	$\mu_{ m pp}$	0.5
Restitution coefficient between particles	$e_{ m pp}$	0.015
Tensile strength	$\sigma_{\scriptscriptstyle \mathrm{T0}}$	50 MPa
Shearing strength	$\sigma_{_{ m S0}}$	125 MPa
Loading rate	U	0.03 m/s

Table 1. Main computational parameters in the DEM simulation

The Poisson's ratio is calculated by choosing 6 equally spaced measurement particles on each vertical plane. The strain in the x and y direction are calculated, and strain in the z direction is determined by the upper and lower boundary displacement of the sample. Hence, the Poisson's

ratio is determined as $\upsilon = -(\Delta \varepsilon_x + \Delta \varepsilon_y)/2\Delta \varepsilon_z$. The relationship between the Poisson's ratio and the strain is shown in Fig 4. The value is relatively small and fluctuates at the initial loading, then stabilizes and the average value is 0.223. The Poisson' ratio in the *x* and *y* direction can also been obtained as 0.220 and 0.226.



Fig.4 Poisson's ratio of rock sample under uniaxial compression

The change of the sample from local damage to final breakage failure is attributed to the accumulation of the bond breakage. Fig 2 also shows the change of number of bond breakage with axial strain. It shows that lots of breakages happen before reaching the maximum stress, then each decrease of the stress accompanies the increase of number of bond breakage. Fig 5 plots the breakage orientation of normal and shearing breakage, in which the angle of normal and shearing force to the horizontal plane correspond to normal and shearing breakage orientation, respectively. The results show tensile breakage orientation centers in the range of $\pm 45^{\circ}$ to the horizontal plane with double peak, whereas shearing breakage orientation centers around $\pm 30^{\circ}$ to the horizontal plane. In this DEM simulation, totally 7082 breakages happen, 1631 are shearing and 5451 are tensile, which indicates that tensile failure is the major failure mode of bonding breakage, accounting for 77% of the total breakage.



Fig.5 Orientation of tensile and shear failures between bonded particles

EFFECT OF PARTICLE SIZE

Keep the sample size constant, and change the average particle size as 6mm, 6.5mm, 7mm, 7.5mm, 8mm, 8.5mm and 9mm to investigate the effect of particle size on the rock failure characteristics under uniaxial compressive. For each particle size, 5 sets of tests are performed and

the corresponding average values are compared. Fig 6 and 7 plot the stress-strain curves, elastic modulus, Poisson's ratio and compressive strength of the rock with particle size of 6mm, 7mm, 8mm and 9mm. The results show that stress-strain curves follow the same trend. The particle size effect on the elastic modulus, Poisson's ratio and compressive strength is not obvious, and the corresponding average values are 42.5GPa, 185.5MPa and 0.22. However, the computation results fluctuate when the ratio of the particle size to the sample size is bigger than 9%.



Fig. 7 Elastic modulus, Poisson's ratio and compressive strength with various particle sizes

CONCLUSION

This paper simulates the uniaxial compression process of rock material using discrete element method with parallel bond. During bonding breakage, tensile breakage accounts for 77% of the total breakage, and the sample finally breaks into two parts, with the inclined plane 45° to the axial direction. The simulation results also show that the compressive strength, elastic modulus and Poisson's ratio of rock material are not sensitive to the change of particle size when the ratio of the particle diameter to the width of the sample is in the range of $6\% \sim 9\%$.

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